J. Fluid Mech. (1965), *vol.* 22, *part* 4, *pp.* 671–677
Printed in Great Britain

Waves in a thin liquid layer on a rotating disk

By HANS ESPIG
Vickers Research Ltd., Sunninghill
AND RUSSELL HOYLE
Department of Engineering Science, University of Durham

(Received 29 December 1964)

Film-thickness measurements on some liquids flowing down vertical surfaces gave values higher than those given by Nusselt's theory. This was shown to be due to the presence of waves on the film surface, and the discrepancy is thought to be due mainly to the omission from Nusselt's theory of the factors causing wave motion. The authors have investigated flow in films of water on a rotating disk to find whether or not this discrepancy between the actual and theoretical thicknesses, as calculated by laminar-flow film theory, occur in this case also.

1. Introduction

The surface of a rotor, such as that of a steam-turbine, can be considered as a series of surfaces rotating at various angles to the axis. The limiting conditions are plain cylinders and disks, being the surfaces situated at zero and ninety degrees to the axis of rotation, respectively. In the case of a steam-turbine rotor, water is being continually added to the surface by condensation and drained from it in two ways. In the case of those parts of the surface similar to a rotating cylinder drainage occurs by drops being thrown off in a direction perpendicular to the surface. In the case of the parts similar to a disk the condensate flows along the surface as a thin film and is ultimately thrown off at the periphery in the form of drops. For other surfaces making angles with the axis between these two limiting conditions the condensate may be drained from the surface in either of these two modes. A water layer draining in a direction perpendicular to the surface has been studied by Hoyle & Matthews (1965) on a rotating cylinder whilst a water layer that is flowing along the surface of a rotating disk is described in this report.

Earlier work on surfaces covered by a liquid film, where drainage occurs along the surface, has been mostly on stationary vertical and inclined plates and tubes. In these stationary cases experimental work has shown that ripples or waves are sometimes observed in the film. Measurements of film thicknesses reported in the literature for condensation and flow of liquid films on a vertical surface are higher than the values predicted from Nusselt's theory (equation (3)). It is thought that this discrepancy is due mainly to the omission from Nusselt's theory of the factors causing wave motion in the condensate film, the other factors omitted from Nusselt's theory having been shown to have only small effects. A series of experiments was therefore carried out by Espig (1964) to find whether or not this discrepancy between the actual film thickness and that predicted by laminar-flow film theory occurred also in the case of a thin liquid film flowing on a horizontal rotating disk.

2. Some earlier work in the same field

2.1. *The onset of wave formation*

For fluids flowing down the walls of a vertical tube, the critical Reynolds number Re at which waves become apparent was determined for a number of fluids by Grimley (1945), who correlated his results with Reynolds numbers calculated from an equation that can be written in the form

$$Re = 1{\cdot}62(\sigma^3\rho/\mu^4 g)^{\frac{1}{8}}, \tag{1}$$

where σ, ρ, μ, and g are surface tension, density, dynamic viscosity, and strength of field of gravity, respectively.

In an analysis by Kapitsa (1948) the Reynolds number for what he described as the transition from laminar to wave flow can be written as

$$Re = 2{\cdot}43(\sigma^3\rho/\mu^4 g)^{\frac{1}{11}}. \tag{2}$$

The values of Reynolds numbers for such an apparent transition were derived by Binnie (1957), Kirkbride (1933), and Friedmann & Miller (1941).

However, Benjamin (1957) argues that there is no transition from plane laminar to wave flow, and that the apparent absence of waves at low Reynolds numbers for films flowing down a vertical plane does not mean there exists a critical value of Re below which uniform laminar flow is entirely stable. In other words, for all finite Reynolds numbers there is a kind of wave-like disturbance that undergoes unbounded amplification according to a linearized theory. His argument shows the amplification to be large only for values of Re greater than about 4. His theory is not incompatible with the experimental work although his deductions are different from those of the experimenters. It is the validity of previous theoretical work that Benjamin challenges.

The experimental work described in this paper, being for Reynolds numbers greater than 20, was carried out on films in which Benjamin and others would agree waves should be clearly observable.

2.2. *The mean thickness of films*

Nusselt (1916) derived an equation for the mean thickness of liquid films flowing down inclined surfaces, which can be written as follows

$$\delta(g/\nu^2)^{\frac{1}{3}} = 0{\cdot}909\, Re^{\frac{1}{3}}, \tag{3}$$

where $Re = (4\dot{Q}/\pi\mu D)$, and δ, ν, and $\dot{Q}$ are the film thickness, kinetic viscosity, and mass flow rate respectively, and $\frac{1}{2}D$ is the distance from the centre of the disk. Kapitsa deduced, from his equation for the thickness of a film under the crests of waves (equation (6)), that the mean film thickness is given by

$$\delta(g/\nu^2)^{\frac{1}{3}} = 0{\cdot}845\, Re^{\frac{1}{3}}. \tag{4}$$

In figure 1 values of the function $\delta(g/\nu^2)^{\frac{1}{3}}$ are plotted against values of Re using equations (3) and (4), and they are compared with experimental values (shown as dots in the figure) derived from the work of others on vertical and inclined planes when waves were not apparent. These include mean film thicknesses

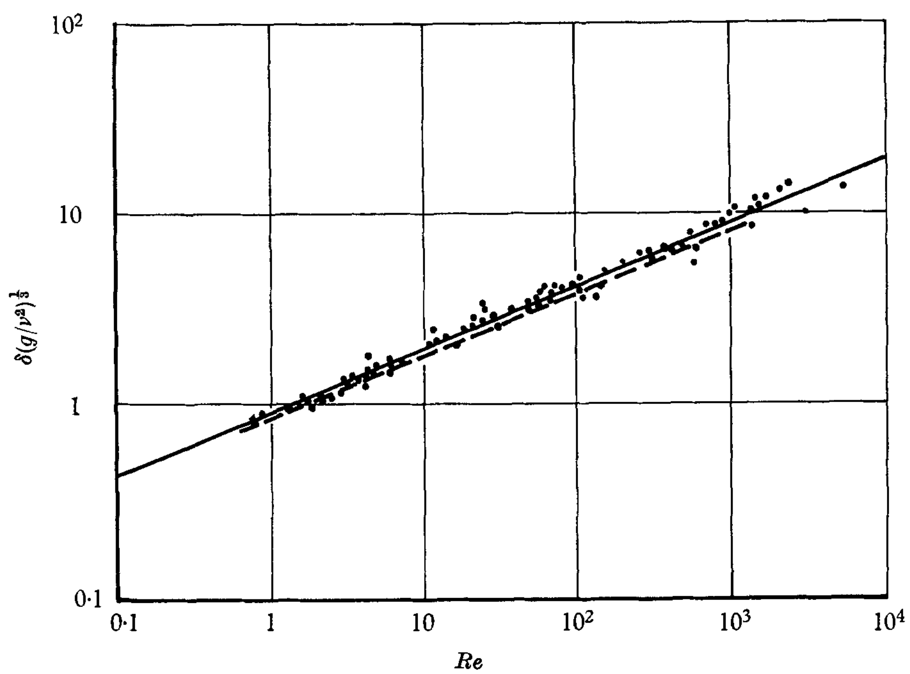

FIGURE 1. Mean film thickness as a function of $Re = (4\dot{Q}/\pi\mu D)$. Theoretical solutions: ——, Nusselt (equations (3) and (5)); — — — —, Kapitsa (equation (4)).

measured by Friedmann & Miller using oil, kerosene, toluene, and water films. It will be seen from these that the experimental values of thickness when waves were not apparent correlated well with mean values calculated from Nusselt's equation (3) and from Kapitsa's equation (4).

In the case of thin films on a disk, rotating at an angular velocity ω, which is another case of drainage of condensate along a surface, $\frac{1}{2}D\omega^2$ should be substituted for g in equation (3) which then becomes

$$\delta(D\omega^2/2\nu^2)^{\frac{1}{3}} = 0{\cdot}909\,Re^{\frac{1}{3}}, \qquad (5)$$

and which should give mean thicknesses of liquid films flowing along the surface of a rotating disk. The curves for this equation are shown in both figures 1 and 3, where they are of course coincident with the curves for Nusselt's equation (3).

2.3. *The thickness beneath the crests of waves*

Equations (3) to (5) in § 2.2 concern either thickness in a condition apparently without waves or mean thicknesses when waves are apparent. Kirkbride, using a micrometer, measured the maximum film thickness for oil and water films flowing down vertical tubes. For Reynolds numbers less than 8, a good correlation was obtained with equation (3) and no waves were observed. For Reynolds numbers between 8 and 1800 the measured film thicknesses were greater than those calculated, and in this range waves were visible on the film surface. Kapitsa deduced that the maximum film thickness was given by

$$\delta(g/\nu^2)^{\frac{1}{3}} = 1{\cdot}232\, Re^{\frac{1}{3}}, \tag{6}$$

which gave values in agreement with Kirkbride's measurements over the range of Reynolds numbers for which Kapitsa's theory was valid. The curve of this equation has been plotted in figure 3 where it is compared with other equations and some experimental work. This equation was also used by Kapitsa to deduce his equation (equation (4)) for mean film thickness by assuming that the amplitude of the waves for all flow rates is equal to 0·46 of the mean film thickness.

It was the authors' intention to deduce an equation similar to equation (6) for the maximum film thickness of the condensate layer on the sides of a rotating disk. Such an equation has been derived experimentally and is discussed in § 4.

3. Experimental work

3.1. *Apparatus*

A diagram of the apparatus used to measure actual film thicknesses on a rotating disk is given in figure 2. Water, delivered to a constant-head tank F, was supplied through the control valve G to the centre of a horizontal disk A by a distributor cup K. The height of the cup above the disk was controlled by adjusting screws P, and both cup and disk were mounted on the spindle of a variable-speed motor E. The water thrown off the periphery of the disk was collected in a tray L and its flow rate was measured by collecting it at the drain M over a known period of time.

A stainless-steel needle probe B, attached to a vernier C, was mounted above the disk to measure the film thickness, and the assembly was supported by the insulated beam N with the needle probe at either of two positions, at 1·5 or at 2·0 in. radius. A dial gauge D mounted on the vernier cross-bar was used to measure the needle movement to 0·0001 in. The film thickness was measured by lowering the probe until it touched the film surface, this being determined by connecting the probe in series with a battery H, a micro-ammeter J, and the constant-head tank, thus completing an electrical circuit through the flowing liquid when the needle touched the film surface. The probe was then lowered until it touched the disk surface, this position being detected by the vibration of the needle. The difference between the two readings on the dial gauge gave the film thickness.

The rotational speed of the disk was measured by a stroboscope which also enabled the film flow conditions to be studied by direct visual observation.

The flow rate of the water was varied between its maximum, giving a Reynolds number of 600, and a minimum when the film broke up into rivulets, which occurred at Reynolds numbers below 10.

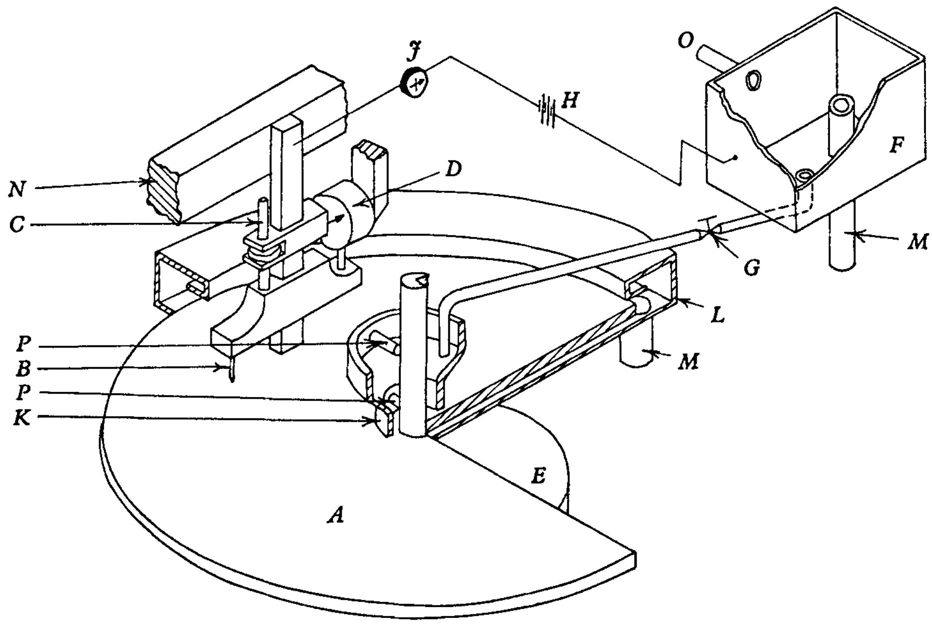

FIGURE 2. Apparatus used to study the flow of a liquid film on a rotating disk. *A*, rotating disk; *B*, needle; *C*, vernier; *D*, dial gauge; *E*, electric motor; *F*, constant-head tank; *G*, valve; *H*, 6 V battery; *J*, micro-ammeter; *K*, distributor cup; *L*, collector tray; *M*, drain; *N*, insulated beam; *O*, water supply; *P*, adjusting screws.

3.2. *Direct visual observation*

Visual examination of the film using a stroboscope indicated that the flow was nearly radial for thin films and that four different types of flow occurred in the film:

(i) The film was, in certain circumstances, broken into rivulets. This occurred when the flow rate was low for the speed of rotation. When laminar flow was present and the film just covered the disk, a slight reduction in flow rate or a slight increase in speed caused the film to break down. Breakdown occurred from the edge in the form of rivulets which ran rapidly towards the centre leaving patches of dry disk, and at the centre a patch of thin film. This patch of thin film remained stationary with respect to the disk but the rivulets draining the patch snaked across the disk surface. Decreasing the speed caused the film to reform again, although once a surface had dried and rivulets formed it was difficult, due to surface tension, to re-establish complete coverage unless the disk was flooded.

(ii) At higher flow rates, a plane uniform film was obtained which appeared to be in laminar flow, although, even at low speeds, low circumferential waves travelling outwards from near the centre to the periphery were sometimes apparent.

(iii) A further small increase in the flow rate caused waves always to be visible on the film surface in the form of circumferential rings moving outward across the disk, the distance between the waves decreasing and the amplitude increasing when the flow rate or the rotational speed was increased.

(iv) At the highest flow rates helical waves were also visible, superimposed on the circumferential waves. If the surface tension was lowered by introducing a detergent, the flow rate required for the onset of visible wave formation was increased. The circumferential waves appeared to be similar to those that occur in a thin film flowing down a vertical tube, and the helical waves similar to the vortices described by Gregory, Stuart, and Walker and referred to by Kirkbride (1933).

In addition to what has been stated in (i) above, decreasing or increasing the disk's speed of rotation appeared to cause an increase or reduction of the wave depth at constant flow rate.

3.3. *Experimental results*

Readings were taken of the circumferential-wave thickness on the disk at 1·5 and 2·0 in. radii for a series of rotational speeds and water flow rates. The needle probe was lowered until the crest of the waves passing across the disk just touched the

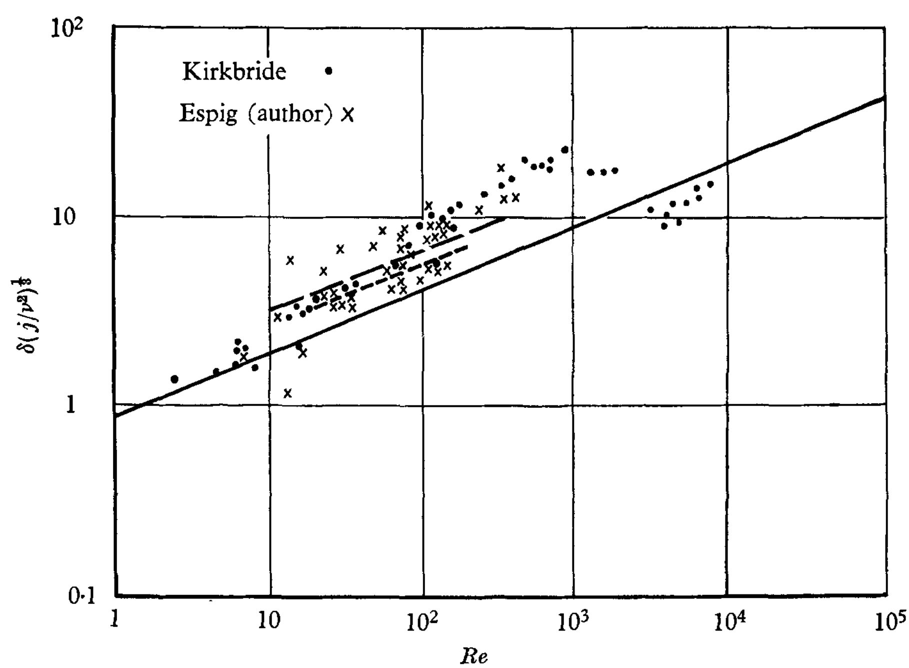

FIGURE 3. Maximum film thickness as a function of *Re*. Theoretical solutions: ——, Nusselt (equations (3) and (5)); – – – – –, Kapitsa (equation (6)); ———, authors (equation (7)).

probe. The readings taken were therefore readings of maximum thickness. Values of $\bar{\delta}(D\omega^2/2\nu^2)^{\frac{1}{3}}$, where the values of $\bar{\delta}$ were those of film thickness beneath the crests of waves, and also of Reynolds number $Re = (4\dot{Q}/\pi\mu D)$ were plotted and are shown as crosses in figure 3.

The measurements made by Kirkbride on thin films flowing down vertical tubes are also shown in the same figure together with the theoretical solutions proposed by Nusselt for plane laminar flow (equation (3)), and by Kapitsa for wave flow (equation (6)). Equation (7), recommended by the authors (see §4) for wave flow on a rotating disk, based on the experimental results, is also shown on the same figure.

4. Discussion

Measurements of the film thickness with the probe when waves were apparent on the film gave the thickness of the film at the crest of each wave and are here called the maximum film thicknesses. The maximum film thickness depends on the radius, speed, and flow rate at which the measurement was taken. As stated above these experimental results have been plotted in figure 3 from which the following conclusions may be drawn.

(i) For the range covered by the experimental work, from $Re = 20$ to $Re = 600$, the maximum film thicknesses measured were greater than the film thickness predicted by equation (5). This is thought to be due to the presence of waves in the film. Equation (5) for films flowing on a rotating disk is the same as Nusselt's equation (equation (3)) for films flowing down a vertical tube, if $\frac{1}{2}D\omega^2$ is substituted for g. The theories on which both these equations are based do not take into account wave formation.

(ii) Kirkbride's measured values, shown by dots in figure 3 are greater than those predicted by Nusselt's equation because the values are for the height of a wave crest and compare well with Kapitsa's equation (6) which also takes account of waves.

(iii) In the case of the rotating disk, to take account of wave formation and therefore to conform more nearly to Espig's experimental values, equation (5) must be written as follows,

$$\bar{\delta}(D\omega^2/2\nu^2)^{\frac{1}{3}} = 1{\cdot}3\, Re^{\frac{1}{3}}. \tag{7}$$

This equation corresponds most nearly to the conditions in the film of water on a rotating disk and is being used by the authors in continued studies of the water films on rotating elements.

REFERENCES

BENJAMIN, BROOKE T. 1957 Wave formation in laminar flow down an inclined plane. *J. Fluid Mech.* **2**, 554.

BINNIE, A. M. 1957 Experiments on onset of wave formation on film of water flowing down vertical plane. *J. Fluid Mech.* **2**, 551.

ESPIG, HANS 1964 Heat transfer by the condensation of steam on a rotating disc. *Ph.D. Thesis, University of London.*

FRIEDMANN, S. J. & MILLER, C. O. 1941 Liquid films in the viscous flow region. *Industr. Engng Chem.* **33**, 885.

GRIMLEY, H. 1945 Liquid flow conditions in packed powers. *Trans. Inst. Chem. Engrs*, **23**, 228.

HOYLE, RUSSELL & MATTHEWS, D. H. 1965 The effect of speed on the condensate layer on a cold cylinder rotating in a steam atmosphere. *J. Fluid Mech.* **22**, 105.

KAPITSA, P. L. 1948 Wave motion of a thin layer of a viscous liquid. *J. Exp. Theor. Phys. U.S.S.R.*, **1**.

KIRKBRIDE, C. G. 1933 Heat transfer by condensing vapour on vertical tubes. *Trans. Amer. Inst. Chem. Engng*, **30**, 170.

NUSSELT, W. 1916 *Z. Ver Deut. Ing.* **60**, 541.